

12.5 | 연습문제 정답

01. (a) 8π (b) $\frac{10\sqrt{5}}{3\sqrt{17}}\pi$ (c) π

02. $\int_0^{\frac{\pi}{2}} \int_0^{2\cos\theta} \int_0^{r^2\cos\theta\sin\theta} r dz dr d\theta = \frac{2}{3}$

03. (a) $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^4 \rho^2 \sin\phi d\rho d\phi d\theta = \frac{64(2-\sqrt{2})}{3}\pi$

(b) $\int_0^{2\pi} \int_0^{\frac{\pi}{6}} \int_0^1 \rho^2 \sin\phi d\rho d\phi d\theta = \frac{2-\sqrt{3}}{3}\pi$

(c) $\int_0^{2\pi} \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \int_0^a \rho^2 \sin\phi d\rho d\phi d\theta = \frac{2}{3}\pi a^3$

04. $8 \int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_{\csc\phi}^2 \rho^2 \sin\phi d\rho d\phi d\theta = \frac{20\sqrt{3}}{3}\pi$

05. $\int_0^{\pi} \int_0^{\sin\theta} (r\sin\theta - r^2) r dr d\theta = \frac{3\pi}{96}$

06. $8\sqrt{3}\pi$

07. (a) $\int_0^{\frac{\pi}{2}} \int_0^2 \int_0^2 r^2 dz dr d\theta = \frac{8}{3}\pi$

(b) $\int_0^{\frac{\pi}{2}} \int_0^2 \int_0^{\sqrt{4-r^2}} \frac{2z}{r} r dz dr d\theta = \frac{8}{3}\pi$

08. $\int_0^{2\pi} \int_0^\pi \int_0^3 \rho^5 \sin\phi \, d\rho \, d\phi \, d\theta = 486\pi$

09. $4 \int_0^{\frac{\pi}{2}} \int_2^3 r \sqrt{9-r^2} \, dr \, d\theta = \frac{10\sqrt{5}}{3}\pi$

10. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^2 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta + \int_0^{2\pi} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{2\sqrt{2}\cos\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$
 $= \frac{16}{3}\pi \left(1 - \frac{1}{\sqrt{2}}\right) - 2\sqrt{2}\pi$

11. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_{\sec\phi}^{2\sec\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta = \frac{7}{3}\pi$

12. (i) 원주좌표계 $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^1 \int_{\sqrt{3}r}^{\sqrt{4-r^2}} r \, dz \, dr \, d\theta = \frac{8-4\sqrt{3}}{3}\pi$

(ii) 구면좌표계 $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\frac{\pi}{6}} \int_0^2 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta = \frac{8-4\sqrt{3}}{3}\pi$

13. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{2\cos\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta = \pi$

14. 4π